

WACE Mathematics Methods ATAR Course



EP Curriculum Map

Please note that EP does not currently provide all necessary resources to meet the current WACE Mathematics Methods study design. Any specific content that is not currently covered by EP is highlighted in yellow.

Unit 1

Topic 1.1: Counting and probability

Combinations

Specific Expectations	Lessons
1.1.1 understand the notion of a combination as a set of r objects taken from a set of n distinct objects	<ul style="list-style-type: none">• Combinations and Permutations• Pascal's Triangle and n Choose r• Binomial Expansion
1.1.2 use the notation $n r$ and the formula $n r = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects	
1.1.3 investigate Pascal's triangle and its properties to link $n r$ to the binomial coefficients of the expansion of $(x+y)^n$ for small positive integers n	

Language of events and sets and Review of the fundamentals of probability

Specific Expectations	Lessons
1.1.4 review the concepts and language of outcomes, sample spaces, and events, as sets of outcomes	<ul style="list-style-type: none"> • Probability Terms and Concepts • Terminology • Outcomes • Likelihood • Venn Diagrams • Using Venn Diagrams • Relative Frequencies • Using Relative Frequencies • Multiplication & Addition Rules
1.1.5 use set language and notation for events, including: a) A (or A') for the complement of an event A b) $A \cap B$ and $A \cup B$ for the intersection and union of events A and B respectively c) $A \cap B \cap C$ and $A \cup B \cup C$ for the intersection and union of the three events A, B and C respectively d) recognise mutually exclusive events.	
1.1.6 use everyday occurrences to illustrate set descriptions and representations of events and set operations	
1.1.7 review probability as a measure of 'the likelihood of occurrence' of an event	
1.1.8 review the probability scale: $0 \leq P(A) \leq 1$ for each event A , with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty	
1.1.9 review the rules: $P(A') = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
1.1.10 use relative frequencies obtained from data as estimates of probabilities	

Conditional probability and independence

Specific Expectations	Lessons
1.1.11 understand the notion of a conditional probability and recognise and use language that indicates conditionality	<ul style="list-style-type: none"> • Introduction to Conditional Probability • Investigating Conditional Probability with Venn Diagrams • Investigating Conditional Probability with Two-Way Tables • Calculating Conditional Probability Using Tree Diagrams • Calculating Conditional Probabilities using Arrays • Word Problems • Introduction to Independence • Investigating Independent Events using Chance Diagrams • Relative Frequencies, and Conditional Probability and Independent Probability • Question Bank: Counting and Probability • Question Bank: Counting and Probability
1.1.12 use the notation $P(B A)$ and the formula $P(A \cap B) = P(A)P(B A)$	
1.1.13 understand the notion of independence of an event A from an event B , as defined by $P(B A) = P(B)$	
1.1.14 establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B , and recognise the symmetry of independence	
1.1.15 use relative frequencies obtained from data as estimates of conditional probabilities and as indications of possible independence of events	

Topic 1.2: Functions and graphs

Lines and linear relationships

Specific Expectations	Lessons
1.2.1 recognise features of the graph of $y=mx+c$, including its linear nature, its intercepts and its slope or gradient	<ul style="list-style-type: none"> • Plotting Linear Equations Using Tables • Drawing the Line from an Equation • Slope and Intercept from a Graph • Equations From Graphs • How to Model Situations • Modelling Situations: Global Warming • Modelling Situations: Gym Membership • Modelling Situations: The Leaky Bike Tyre • Modelling Situations: The Road Trip
1.2.2 determine the equation of a straight line given sufficient information; including parallel and perpendicular lines	
	Revision: Solving Linear Equations <ul style="list-style-type: none"> • Solving Linear Equations with Fractions • Using Graphs to Solve Simultaneous Equations • Using Elimination to Solve Simultaneous Equations • Using Substitution to Solve Simultaneous Equations • Non-Linear Simultaneous Equations

Quadratic relationships

Specific Expectations	Lessons
1.2.3 examine examples of quadratically related variables	<ul style="list-style-type: none"> • Parabolas • Parabola Transformations • Multiple Transformations of Parabolas • Monic Factorisation • Non-Monic Factorisation • Solving Monic Quadratic Equations • Solving Non-Monic Quadratic Equations • The Quadratic Formula
1.2.4 recognise features of the graphs of $y=x^2$, $y=a(x-b)^2+c$, and $y=ax^2+bx+c$, including their parabolic nature, turning points, axes of symmetry and intercepts	
1.2.5 solve quadratic equations, including the use of quadratic formula and completing the square	
1.2.6 determine the equation of a quadratic given sufficient information	
1.2.7 determine turning points and zeros of quadratics and understand the role of the discriminant	
1.2.8 recognise features of the graph of the general quadratic $y=ax^2+bx+c$	

Inverse proportion

Specific Expectations	Lessons
1.2.9 examine examples of inverse proportion	<ul style="list-style-type: none"> • Inverse Proportion • Hyperbola Graphs • Hyperbola Graph Transformations • Inverse Functions and Transformations • Applications of Quadratic Equations
1.2.10 recognise features and determine equations of the graphs of $y=1/x$ and $y=ax-b$, including their hyperbolic shapes and their asymptotes.	

Powers and polynomials

Specific Expectations	Lessons
1.2.11 recognise features of the graphs of $y=x^n$ for $n \in \mathbb{N}$, $n=-1$ and $n=1/2$, including shape, and behaviour as $x \rightarrow \infty$ and $x \rightarrow -\infty$	<ul style="list-style-type: none"> • Introduction to Polynomials • Operating with Polynomials • Evaluating Polynomials • Expanding Quadratic and Cubic Expressions • Factorising Cubic Polynomials • The Factor Theorem • Solving Polynomials • Solving Equations involving Cubic and Quartic Polynomials • Expanding Cubic Expressions • Features of Polynomial Graphs • Features of Graphs - Roots • Cubics • Cubic Transformations • Cubic Transformations
1.2.12 identify the coefficients and the degree of a polynomial	
1.2.13 expand quadratic and cubic polynomials from factors	
1.2.14 recognise features and determine equations of the graphs of $y=x^3$, $y=a(x-b)^3+c$ and $y=kx-ax-bx-c$, including shape, intercepts and behaviour as $x \rightarrow \infty$ and $x \rightarrow -\infty$	
1.2.15 factorise cubic polynomials in cases where all roots are given or easily obtained from the graph	
1.2.16 solve cubic equations using technology, and algebraically in cases where all roots are given or easily obtained from the graph	

Graphs of relations

Specific Expectations	Lessons
1.2.17 recognise features and determine equations of the graphs of $x^2+y^2=r^2$ and $x-a^2+y-b^2=r^2$, including their circular shapes, their centres and their radii	<ul style="list-style-type: none"> • Circle Graphs • The Graph of $y^2=x$ • Question Bank: Functions and Graphs • Question Bank: Functions and Graphs
1.2.18 recognise features of the graph of $y^2=x$, including its parabolic shape and its axis of symmetry	

Functions

Specific Expectations	Lessons
1.2.19 understand the concept of a function as a mapping between sets and as a rule or a formula that defines one variable quantity in terms of another	<ul style="list-style-type: none"> • Introduction to Functions • Function Notation • Transformations of Functions • Find the Range of a Function
1.2.20 use function notation; determine domain and range; recognise independent and dependent variables	
1.2.21 understand the concept of the graph of a function	
1.2.22 examine translations and the graphs of $y=fx+a$ and $y=f(x-b)$	
1.2.23 examine dilations and the graphs of $y=cfx$ and $y=fdx$	
1.2.24 recognise the distinction between functions and relations and apply the vertical line test	

Topic 1.3: Trigonometric functions

Cosine and sine rules

Specific Expectations	Lessons	
1.3.1 review sine, cosine and tangent as ratios of side lengths in right-angled triangles	<ul style="list-style-type: none"> • Trigonometric Ratios • The Sine Rule • Finding Angles Using the Sine Rule • The Sine Rule: The Ambiguous Case • The Cosine Rule • Finding Angles Using the Cosine Rule • Area of a Triangle: $\frac{1}{2} bc \sin A$ 	<ul style="list-style-type: none"> • Using Trigonometric Rules to Model and Solve Problems • Review Lesson: Trigonometric Rules • Review Lesson: Trigonometric Rules • Extension: Heron's Formula • Special Triangles: 30-60-90 • Special Triangles: 45-45-90 • Trigonometric Ratios and Complementary Angles • Question Bank: Triangle Formulae • Question Bank: Triangle Formulae
1.3.2 understand the unit circle definition of $\cos \theta$, \sin and \tan and periodicity using degrees		
1.3.3 examine the relationship between the angle of inclination of a line and the gradient of that line		
1.3.4 establish and use the cosine and sine rules, including consideration of the ambiguous case and the formula $\text{Area} = \frac{1}{2} bc \sin A$ for the area of a triangle		

Circular measure and radian measure

Specific Expectations	Lessons
1.3.5 define and use radian measure and understand its relationship with degree measure	Introduction to Radians
1.3.6 use radian measure to calculate lengths of arcs and areas of sectors and segments in a circle	The Unit Circle and Radians Finding an Arc Length Area of Sectors and Segments

Trigonometric functions

Specific Expectations	Lessons	
1.3.7 understand the unit circle definition of $\sin \theta$, and and periodicity using radians	<ul style="list-style-type: none"> • Understanding and Graphing Sine • Understanding and Graphing Cosine • Understanding and Graphing Tangent • Comparing Trigonometric Functions • Investigating the Effect of Parameters on Trigonometric Graphs • Sketching Transformed Trigonometric Graphs 	<ul style="list-style-type: none"> • Using Trigonometric Functions to Solve Practical Problems • Forestry Subdivision • Balloons Over Waikato • The Pythagorean Identity • Solving equations involving trigonometric functions • Question Bank: Trigonometric Functions 1 • Question Bank: Trigonometric Functions 1
1.3.8 recognise the exact values of $\sin \theta$, and at integer multiples of π and $\frac{\pi}{2}$		
1.3.9 recognise the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$ on extended domains		
1.3.10 examine amplitude changes and the graphs of $y = a \sin x$ and $y = a \cos x$		
1.3.11 examine period changes and the graphs of $y = \sin bx$, $y = \cos bx$ and $y = \tan bx$		
1.3.12 examine phase changes and the graphs of $y = \sin(x-c)$, $y = \cos(x-c)$ and $y = \tan(x-c)$		
1.3.13 examine the relationships $\sin(x + \frac{\pi}{2}) = \cos x$ and $\cos(x - \frac{\pi}{2}) = \sin x$		
1.3.14 prove and apply the angle sum and difference identities		
1.3.15 identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems		
1.3.16 solve equations involving trigonometric functions using technology, and algebraically in simple cases		

Unit 2

Topic 2.1: Exponential functions

Indices and the index laws

Specific Expectations	Lessons
2.1.1 review indices (including fractional and negative indices) and the index laws	<ul style="list-style-type: none"> • Ordering Numbers and Estimating Calculations in Scientific Notation (Standard Form) • Adding and Subtracting with Scientific Notation (Standard Form) • Multiplying and Dividing in Scientific Notation (Standard Form) • Significant Figures and Scientific Notation (Standard Form)
2.1.2 use radicals and convert to and from fractional indices	
2.1.3 understand and use scientific notation and significant figures	
	<ul style="list-style-type: none"> • Introduction to Surds • Index Laws and Fractional Powers • Introduction to Scientific Notation (Standard Form) - Large Numbers • Introduction to Scientific Notation (Standard Form) - Small Numbers • Definitions List: Scientific Notation • Question Bank: Exponential Functions 1 • Question Bank: Exponential Functions 1

Exponential functions

Specific Expectations	Lessons
2.1.4 establish and use the algebraic properties of exponential functions	<ul style="list-style-type: none"> • Question Bank: Exponential Functions 2 • Question Bank: Exponential Functions 2 • Solving Exponential Equations • Exponential Graphs • Features of Exponential Graphs • Using Exponential Functions to Solve Practical Problems
2.1.5 recognise the qualitative features of the graph of $y=ax$ ($a>0$), including asymptotes, and of its translations ($y=ax+b$ and $y=ax-c$)	
2.1.6 identify contexts suitable for modelling by exponential functions and use them to solve practical problems	
2.1.7 solve equations involving exponential functions using technology, and algebraically in simple cases	

Topic 2.2: Arithmetic and geometric sequences and series

Arithmetic sequences

Specific Expectations	Lessons	
2.2.1 recognise and use the recursive definition of an arithmetic sequence: $t_{n+1}=t_n+d$	<ul style="list-style-type: none">• Introduction to Arithmetic Sequences• Recursive Arithmetic Sequences• Finding an Arithmetic Term• Finding a Term Number for an Arithmetic Sequence• Sequences and Series Using Technology• Using Arithmetic Sequences to Model and Analyse Practical Situations	<ul style="list-style-type: none">• Sigma Notation• The Arithmetic Sum Rule• Arithmetic Sums: Solving for the First Term or Common Difference• Partial Sums of Arithmetic Sequences• Solving for an Arithmetic Term Number• Review: Arithmetic Sequences• Review: Arithmetic Sequences
2.2.2 develop and use the formula $t_n=t_1+n-1d$ for the general term of an arithmetic sequence and recognise its linear nature		
2.2.3 use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest		
2.2.4 establish and use the formula for the sum of the first n terms of an arithmetic sequence		

Geometric sequences

Specific Expectations	Lessons	
2.2.5 recognise and use the recursive definition of a geometric sequence: $t_{n+1}=t_n r$		
2.2.6 develop and use the formula $t_n=t_1 r^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature		
2.2.7 understand the limiting behaviour as $n \rightarrow \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r		
2.2.8 establish and use the formula $S_n=t_1 r^n - 1/r - 1$ for the sum of the first n terms of a geometric sequence		
2.2.9 use geometric sequences in contexts involving geometric growth or decay, such as compound interest	<ul style="list-style-type: none"> • Geometric Sequences • Recursive Geometric Sequences • Summing Geometric Sequences • Logs of Geometric Sequences and Sums • Sums to Infinity • Using Geometric Sequences to Model and Analyse Practical Problems • Review: Geometric Sequences <p>Mixed Sequences</p> <ul style="list-style-type: none"> • Mixed Sequences and Series: Finding the Value of a Term • Mixed Sequences and Series: Finding a Term Number 	<ul style="list-style-type: none"> • Mixed Sequences and Series: Finding a Sum of Values • Using Sequences and Series in Context • Alice's New Car • Hamilton's Frogs • Problem Solving: Cold Case • Student Accommodation • Fibonacci Sequence • Graphing Sequences • Question Bank: Arithmetic and Geometric Sequences and Series 2 • Question Bank: Arithmetic and Geometric Sequences and Series 2

Topic 2.3: Introduction to differential calculus

Rates of change

Specific Expectations	Lessons
2.3.1 interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f	<ul style="list-style-type: none"> • Rates of Change • Applications of Rates of Change
2.3.2 use the Leibniz notation δx and δy for changes or increments in the variables x and y	
2.3.3 use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y=f(x)$	
2.3.4 interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y=f(x)$	

The concept of the derivative

Specific Expectations	Lessons
2.3.5 examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \rightarrow 0$ as an informal introduction to the concept of a limit	<ul style="list-style-type: none"> • Introduction to Derivatives • Sketching the Gradient Function from the Original Function • Differentiation By First Principles • Review: Introduction to Derivatives • Features of Graphs
2.3.6 define the derivative $f'(x)$ as $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$	
2.3.7 use the Leibniz notation for the derivative: $\frac{dy}{dx} = \frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx} = f'(x)$ where $y=f(x)$	
2.3.8 interpret the derivative as the instantaneous rate of change	
2.3.9 interpret the derivative as the slope or gradient of a tangent line of the graph of $y=f(x)$	

Computation and Properties of Derivatives

Specific Expectations	Lessons
2.3.10 estimate numerically the value of a derivative for simple power functions	<ul style="list-style-type: none"> • Establishing the Formula for Differentiating Polynomials • Differentiation of Polynomials • Differentiating Polynomials • Rearranging Expressions to Index Form • Rearranging into Index Form with Negative and Non-Integer Powers • Differentiating Negative Powers • Differentiating Non-Integer Powers • Review Lesson: Differentiating Polynomials
2.3.11 examine examples of variable rates of change of non-linear functions	
2.3.12 establish the formula $\frac{d}{dx}x^n = nx^{n-1}$ for non-negative integers n expanding $(x+h)^n$ or by factorising $(x+h)^n - x^n$	
2.3.13 understand the concept of the derivative as a function	
2.3.14 identify and use linearity properties of the derivative	
2.3.15 calculate derivatives of polynomials	

Applications of derivatives

Specific Expectations	Lessons
2.3.16 determine instantaneous rates of change	<p>Rectilinear Motion</p> <ul style="list-style-type: none"> • Distance, Velocity and Acceleration • Kinematics <p>Graphing and Derivatives</p> <ul style="list-style-type: none"> • Plotting and Reading Travel Graphs • Analysing Travel Graphs • Finding a Tangent to a Curve • Finding a Normal to a Curve • Review Lesson: Tangents and Normals • Finding Stationary Points • Classifying Stationary Points by Reading Graphs <p>Applications of Derivatives</p> <ul style="list-style-type: none"> • Increasing and Decreasing Functions • Sketching Graphs • Review Lesson: Stationary Points • Questions on Differentiation Rules • Optimisation • Differentiating Polynomials: River Float • Practice Assessment - Bridge Construction • Rates of Change: Zeppelins
2.3.17 determine the slope of a tangent and the equation of the tangent	
2.3.18 construct and interpret position-time graphs with velocity as the slope of the tangent	
2.3.19 recognise velocity as the first derivative of displacement with respect to time	
2.3.20 sketch curves associated with simple polynomials, determine stationary points, and local and global maxima and minima, and examine behaviour as $x \rightarrow \infty$ and $x \rightarrow -\infty$	
2.3.21 solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains	

Anti-derivatives

Specific Expectations	Lessons
2.3.22 calculate anti-derivatives of polynomial functions	<ul style="list-style-type: none">• Sketching the Original Function• Anti-Differentiating Polynomials• Equation of the Original Function• Question Bank: Introduction to Differential Calculus• Question Bank: Introduction to Differential Calculus

Unit 3

Topic 3.1: Further differentiation and applications

Exponential functions

Specific Expectations	Lessons
3.1.1 estimate the limit of $ah^{-1}h$ as $h \rightarrow 0$, using technology, for various values of $a > 0$	<ul style="list-style-type: none">• Defining the Exponential Function• Differentiating Exponential Functions• Modelling with Derivatives of Exponential Functions
3.1.2 identify that e is the unique number a for which the above limit is 1	
3.1.3 establish and use the formula $\frac{d}{dx}e^x = e^x$	
3.1.4 use exponential functions of the form Ae^{kx} and their derivatives to solve practical problems	

Trigonometric functions

Specific Expectations	Lessons
3.1.5 establish the formulas $\frac{d}{dx}\sin x = \cos x$ and $\frac{d}{dx}\cos x = -\sin x$ by graphical treatment, numerical estimations of the limits, and informal proofs based on geometric constructions	<ul style="list-style-type: none">• Questions on Trigonometric Functions• Establishing the Derivatives of Sine and Cosine• Differentiation of the Sine and Cosine Functions• Applications of Derivatives of Trigonometric Functions
3.1.6 use trigonometric functions and their derivatives to solve practical problems	

Differentiation rules

Specific Expectations	Lessons
3.1.7 examine and use the product and quotient rules	<ul style="list-style-type: none"> • The Product Rule • The Quotient Rule • The Chain Rule
3.1.8 examine the notion of composition of functions and use the chain rule for determining the derivatives of composite functions	
3.1.9 apply the product, quotient and chain rule to differentiate functions such as x^e , $\tan x$, $1/x^n$, $x \sin x$, $e^x \sin x$ and $a^x - b$	

The second derivative and applications of differentiation

Specific Expectations	Lessons
3.1.10 use the increments formula: $\delta y = dy/dx \times \delta x$ to estimate the change in the dependent variable y resulting from changes in the independent variable x	<ul style="list-style-type: none"> • Understanding The Second Derivative • The Second Derivative • Questions - The 2nd derivative and applications • Using the Second Derivative to Find Local Maxima and Minima • Sketching the Graph of a Function Using Derivatives • Solving Optimisation Problems Given a Function • Solving Optimisation Problems by Developing a Function • Question Bank - Further Differentiation and Applications 3 • Question Bank - Topic 3.1: Further Differentiation and Applications • Question Bank: Further Differentiation and Applications 3
3.1.11 apply the concept of the second derivative as the rate of change of the first derivative function	
3.1.12 identify acceleration as the second derivative of position with respect to time	
3.1.13 examine the concepts of concavity and points of inflection and their relationship with the second derivative	
3.1.14 apply the second derivative test for determining local maxima and minima	
3.1.15 sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection	
3.1.16 solve optimisation problems from a wide variety of fields using first and second derivatives	

Topic 3.2: Integrals

Anti-differentiation

Specific Expectations	Lessons	
3.2.1 identify anti-differentiation as the reverse of differentiation	<ul style="list-style-type: none">• Integrating Polynomials• Integrating Rational Functions and Surds• Finding the Constant of Integration• Integrating Exponentials• Integrating Trigonometric Functions• Trigonometric Functions: Finding the Constant• Integrating Products: Reverse Chain Rule• Integration of Inverses• Integrating Quotients: Splitting Fractions	<ul style="list-style-type: none">• Integrating Quotients: Factorising and Cancelling• Integrating Sums & Transformations of Functions• Determining the Original Function given the Derivative and an Initial Condition• Determining Integrals by Recognition• Extension: Integration by Substitution• Extension: Identifying Substitutions• Further Anti-Differentiation of Polynomials
3.2.2 use the notation $\int f(x) dx$ for anti-derivatives or indefinite integrals		
3.2.3 establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$		
3.2.4 establish and use the formula $\int e^x dx = e^x + c$		
3.2.5 establish and use the formulas $\int \sin x dx = -\cos x + c$ and $\int \cos x dx = \sin x + c$		
3.2.6 identify and use linearity of anti-differentiation		
3.2.7 determine indefinite integrals of the form $\int (ax + b) dx$		
3.2.8 identify families of curves with the same derivative function		
3.2.9 determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$		

Definite integrals and Fundamental theorem

Specific Expectations	Lessons
3.2.10 examine the area problem and use sums of the form $\sum_{i=1}^n f(x_i) \delta x_i$ to estimate the area under the curve $y=f(x)$	<ul style="list-style-type: none"> • Introduction to Definite Integrals • Calculating Definite Integrals • Definite Integrals: Further Questions • Anti-differentiation • Anti-differentiation - Area between curves
3.2.11 identify the definite integral $\int_a^b f(x) dx$ as a limit of sums of the form $\sum_{i=1}^n f(x_i) \delta x_i$	
3.2.12 interpret the definite integral $\int_a^b f(x) dx$ as area under the curve $y=f(x)$ if $f(x) > 0$	
3.2.13 interpret $\int_a^b f(x) dx$ as a sum of signed areas	
3.2.14 apply the additivity and linearity of definite integrals	
3.2.15 examine the concept of the signed area function $F(x) = \int_a^x f(t) dt$	
3.2.16 apply the theorem: $F'(x) = \frac{d}{dx} \int_a^x f(t) dt = f(x)$, and illustrate its proof geometrically	
3.2.17 develop the formula $\int_a^b f'(x) dx = f(b) - f(a)$ and use it to calculate definite integrals	

Applications of integration

Specific Expectations	Lessons
3.2.18 calculate total change by integrating instantaneous or marginal rate of change	<ul style="list-style-type: none"> • Kinematics • Displacements and Velocities Given Acceleration and Initial Values • Determining Displacement From Velocity • Rates of Change • Calculating Total Change • Approximating the Area Under Curves • The Rectangle Rule • The Trapezium Rule • Simpson's Rule • Finding the Range of Integration
3.2.19 calculate the area under a curve	
3.2.20 calculate the area between curves determined by functions of the form $y=f(x)$	
3.2.21 determine displacement given velocity in linear motion problems	
3.2.22 determine positions given linear acceleration and initial values of position and velocity.	
	<ul style="list-style-type: none"> • Area Under a Curve • Area Above and Below the x-Axis • Area Between Two Curves • Calculating the Area Between Two Curves • Calculating the Area Under a Curve • Area Beneath a Curve: Further Questions • Question Bank: Integrals • Topic 3.2: Integrals • Question Bank: Integrals

Topic 3.3: Discrete random variables

General discrete random variables

Specific Expectations	Lessons
3.3.1 develop the concepts of a discrete random variable and its associated probability function, and their use in modelling data	<ul style="list-style-type: none">● Introduction to Discrete Random Variables● Relative Frequency with Discrete Random Variables● Uniform Discrete Random Variables● Examining Non-uniform Discrete Random Variables● Expected Number● Using Variance and Standard Deviation of Discrete Random Variables● Discrete Random Variables to Solve Practical Problems● Question Bank: Discrete Random Variables 1● Question Bank: Discrete Random Variables 1
3.3.2 use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable	
3.3.3 identify uniform discrete random variables and use them to model random phenomena with equally likely outcomes	
3.3.4 examine simple examples of non-uniform discrete random variables	
3.3.5 identify the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases	
3.3.6 identify the variance and standard deviation of a discrete random variable as measures of spread, and evaluate them using technology	
3.3.7 examine the effects of linear changes of scale and origin on the mean and the standard deviation	
3.3.8 use discrete random variables and associated probabilities to solve practical problems	

Bernoulli distributions and binomial distributions

Specific Expectations	Lessons
3.3.9 use a Bernoulli random variable as a model for two-outcome situations	<ul style="list-style-type: none"> • Bernoulli Distributions • Binomial Distribution • Calculating the Mean and Variance of a Binomial Distribution • Using Binomial Distributions to Model and Solve Practical Problems • Question Bank - Bernoulli Distributions • Question Bank - Binomial Distributions • Question Bank -Topic 3.3: Discrete Random Variables • Question Bank: Discrete Random Variables 2 • Question Bank: Discrete Random Variables 2
3.3.10 identify contexts suitable for modelling by Bernoulli random variables	
3.3.11 determine the mean p and variance $p(1-p)$ of the Bernoulli distribution with parameter p	
3.3.12 use Bernoulli random variables and associated probabilities to model data and solve practical problems	
3.3.13 examine the concept of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial	
3.3.14 identify contexts suitable for modelling by binomial random variables	
3.3.15 determine and use the probabilities $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$ associated with the binomial distribution with parameters n and p ; note the mean np and variance $np(1-p)$ of a binomial distribution	
3.3.16 use binomial distributions and associated probabilities to solve practical problems	

Unit 4

Topic 4.1: The logarithmic function

Logarithmic functions

Specific Expectations	Lessons
4.1.1 define logarithms as indices: $ax=b$ is equivalent to $x=b$ i.e. $ab =b$	<ul style="list-style-type: none">• Introduction to Logarithms• Solving Simple Logarithmic Equations• Deriving the Laws of Logarithms• The Logarithm Laws• Combining the Logarithm Laws• Deriving the Laws of Logarithms• Establish and Use Logarithmic Laws and Definitions• Solving Using the Logarithm Laws• Combining the Logarithm Laws <ul style="list-style-type: none">• Solving Exponential Equations Using Logarithms• Solving Equations Involving Logarithmic Functions• Applications of Exponential Equations• Exponentials and Logarithms: Multiple Variables• Logarithmic Scales• Interpreting and Using Logarithmic Scales• Features of Logarithmic Graphs• Question Bank: The Logarithmic Function 1• Question Bank: The Logarithmic Function 1• Question Bank: The Logarithmic Function 2• Question Bank: The Logarithmic Function 2
4.1.2 establish and use the algebraic properties of logarithms	
4.1.3 examine the inverse relationship between logarithms and exponentials: $y=ax$ is equivalent to $x=y$	
4.1.4 interpret and use logarithmic scales	
4.1.5 solve equations involving indices using logarithms	
4.1.6 identify the qualitative features of the graph of $y=x$ ($a>1$), including asymptotes, and of its translations $y=x +b$ and $(x-c)$	
4.1.7 solve simple equations involving logarithmic functions algebraically and graphically	
4.1.8 identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems	

Calculus of the natural logarithmic function

Specific Expectations	Lessons
4.1.9 define the natural logarithm $\ln x = x$	<ul style="list-style-type: none"> • Differentiating Exponential Functions with Different Bases • The Natural Logarithm and Inverse Relations • Differentiating the Natural Logarithm • Using Logarithmic Functions and Their Derivatives to Solve Problems • Problem Solving with Derivatives of Natural Logarithms • Deriving Logarithms of Base a • Question Bank - Topic 4.1: The Logarithmic Function
4.1.10 examine and use the inverse relationship of the functions $y=e^x$ and $y=\ln x$	
4.1.11 establish and use the formula $\frac{d}{dx} \ln x = \frac{1}{x}$	
4.1.12 establish and use the formula $\int \frac{1}{x} dx = \ln x + c$, for $x > 0$	
4.1.13 determine derivatives of the form $\frac{d}{dx} \ln f(x)$ and integrals of the form $\int f'(x) f(x) dx$, for $f(x) > 0$	
4.1.14 use logarithmic functions and their derivatives to solve practical problems	

Topic 4.2: Continuous random variables and the normal distribution

General continuous random variables

Specific Expectations	Lessons
4.2.1 use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable	<ul style="list-style-type: none"> • Continuous Random Variables • Calculating the Expected Value, Standard Deviation and Variance of Continuous Random Variables • Questions on General Continuous Random Variables
4.2.2 examine the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts	
4.2.3 identify the expected value, variance and standard deviation of a continuous random variable and evaluate them using technology	
4.2.4 examine the effects of linear changes of scale and origin on the mean and the standard deviation	

Normal distributions

Specific Expectations	Lessons	
4.2.5 identify contexts, such as naturally occurring variation, that are suitable for modelling by normal random variables	<ul style="list-style-type: none"> • Topic 4.2: Continuous Random Variables and the Normal Distribution • Normal Random Variables • Introducing the Normal Distribution • The Standard Normal Distribution • Calculating Probabilities with the Normal Distribution • Using Normal Distributions to Model and Solve Practical Problems • Applications of the Normal Distribution 	<ul style="list-style-type: none"> • Working Backwards: Calculating Bounds • Working Backwards: Mean and Standard Deviation • The Normal Distribution: Further Questions • Question Bank: Continuous Random Variables and The Normal Distribution • Question Bank: Continuous Random Variables and The Normal Distribution • Questions - General Continuous Random Variables
4.2.6 identify features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution		
4.2.7 calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve practical problems		

Topic 4.3: Interval estimates for proportions

Random sampling

Specific Expectations	Lessons
4.3.1 examine the concept of a random sample	<ul style="list-style-type: none"> • Types of Sampling: Probability Sampling • Types of Sampling: Non-Probability Sampling • Sampling Errors • Analysing Sampling in Reports • Misleading Reports • Questions - Random Sampling
4.3.2 discuss sources of bias in samples, and procedures to ensure randomness	
4.3.3 use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli	

Sample proportions

Specific Expectations	Lessons
4.3.4 examine the concept of the sample proportion p as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{p(1-p)}$ of the sample proportion p	<ul style="list-style-type: none">• Introduction to Random Sampling and Bias• Investigating the Variability of Random Samples• Sample proportions, means and standard deviation• Approximating and Simulating the Distribution of Sample Proportions
4.3.5 examine the approximate normality of the distribution of p for large samples	
4.3.6 simulate repeated random sampling, for a variety of values of p and a range of sample sizes, to illustrate the distribution of p and the approximate standard normality of $\sqrt{p(1-p)}$ where the closeness of the approximation depends on both n and p	

Confidence intervals for proportions

Specific Expectations	Lessons
4.3.7 examine the concept of an interval estimate for a parameter associated with a random variable	<ul style="list-style-type: none">• Question Bank: Interval Estimates for Proportions• Question Bank: Interval Estimates for Proportions• Interval Estimates & Confidence Intervals• Approximating and Simulating Margins of Error and Levels of Confidence• Questions - Confidence Intervals• Topic 4.3: Interval Estimates for Proportions
4.3.8 use the approximate confidence interval $p \pm z\sqrt{p(1-p)}$ as an interval estimate for p , where z is the appropriate quantile for the standard normal distribution	
4.3.9 define the approximate margin of error $E = z\sqrt{p(1-p)}$ and understand the trade-off between margin of error and level of confidence	
4.3.10 use simulation to illustrate variations in confidence intervals between samples and to show that most, but not all, confidence intervals contain p	